

### 8.2.1 Kinetic model for aerobic sludge digestion

The model for aerobic sludge digestion is based on the following fundamental concepts:

- (1) Only the active sludge concentration is susceptible to aerobic digestion and the decay rate of this active sludge is proportional to its concentration. The digestion process does not affect the inactive sludge fractions. The decay rate of the active sludge can be expressed as:

$$r_d = (dX_a/dt)_d = -b_h \cdot X_a \quad (8.5)$$

Where:

$r_d$  = active sludge decay rate

- (2) Only part of the decayed active sludge is oxidised into inorganic products. The remaining fraction “f” accumulates as an inactive organic solid in the mixed liquor, which is called the endogenous residue:

$$(dX_e/dt)_d = -f \cdot (dX_a/dt)_d = -f \cdot r_d \quad (8.6)$$

The existence of the endogenous residue has been demonstrated by Washington and Hetling (1962), who noted a linear increase in the sludge concentration in an activated sludge process, when this was fed with a soluble biodegradable substrate and without sludge discharge. It was assumed that the biodegradable influent material is metabolised and the active sludge thus generated decays to produce the endogenous residue, resulting in the increase of the sludge concentration.

The observed increase of the sludge mass was 0.09 mg VSS.mg<sup>-1</sup> COD applied. Considering that the active sludge production Y equals 0.45 mg active sludge per mg metabolised COD, it can be calculated that a fraction  $f = 0.09/0.45 = 0.2$  of the decayed active sludge remains as the endogenous residue. Other research workers (Brodersen and McCarty, 1964; Marais and Ekama, 1976 and Dias et al, 1981) found similar values for f, independent from operational parameters such as sludge age, temperature and stabilised sludge composition.

Having established the value of the endogenous fraction “f”, the constant “b<sub>h</sub>” must be determined experimentally to define the kinetic model for aerobic digestion. This can be done conveniently by observing the aerobic digestion of activated sludge batches. In a batch reactor the decrease of the active sludge concentration in time can be calculated by integration of Eq. (8.5):

$$X_a = X_{ai} \cdot \exp(-b_h \cdot t) \quad (8.7)$$

Where:

$X_{ai}$  = initial active sludge concentration

t = aerobic digestion time

As the active sludge concentration cannot be determined experimentally, the validity of Eq. (8.7) cannot be established directly and must be confirmed by the behaviour of the parameters that are affected by aerobic digestion. The relationship between the active sludge concentration and the value of these parameters will now be derived:

### 8.2.1.1 Variation of the volatile sludge concentration

The variation of the volatile sludge concentration is equal to the decrease of the active sludge concentration and is partly compensated by the increase of the endogenous residue, which amounts to a fraction “f” of the decayed active sludge concentration. Hence:

$$X_{vi} - X_v = (X_{ai} - X_a) + X_e = (X_{ai} - X_a) + f \cdot (X_{ai} - X_a) \quad (8.8)$$

Where:

$X_{vi}$  = initial volatile sludge concentration

$X_e$  = concentration of endogenous residue generated in the sludge batch

Substituting Eq. (8.7) in Eq. (8.8) one has:

$$X_v = X_{vi} - (1 - f) \cdot X_{ai} \cdot (1 - \exp(-b_h \cdot t)) \quad (8.9)$$

When all active sludge has decayed, the volatile sludge concentration has reached a stable minimum value, which can be calculated as:

$$X_{v\infty} = X_{vi} - (1 - f) \cdot X_{ai} \quad (8.10)$$

Where:

$X_{v\infty}$  = volatile sludge concentration after completion of the active sludge decay.

Now, the difference of the sludge concentration after a time “t” of digestion and after complete digestion is expressed as:

$$X_v - X_{v\infty} = (1 - f) \cdot X_{ai} \cdot \exp(-b_h \cdot t) \text{ or}$$

$$\log (X_v - X_{v\infty}) = \log [(1 - f) \cdot X_{ai}] - 2.3 \cdot b_h \cdot t \quad (8.11)$$

Equation (8.11) can be used to determine the constant  $b_h$  with the following experimental procedure:

- (1) Determine the volatile sludge concentration in a digesting sludge batch as a function of time, until a constant value has been reached (this will take a few weeks);
- (2) Plot the experimental values of  $X_v - X_{v\infty}$  as function of the digestion time on semilog paper;
- (3) The best-fit straight line through the experimental data will have a gradient equal to  $-2.3 \cdot b_h$ .

### 8.2.1.2 Variation of the oxygen uptake rate

An alternative method to determine the value of  $b_h$  is by measuring the oxygen uptake rate (OUR) of a batch of digesting sludge. Since the oxidation of (1 - f) gram of volatile solids (originating from the decay of 1 gram of active sludge) requires an oxygen mass of  $f_{cv} \cdot (1 - f)$  gram  $O_2$ , the oxygen uptake rate is directly linked to the decay rate:

$$O_c = f_{cv} \cdot (1 - f) \cdot r_d = f_{cv} \cdot (1 - f) \cdot b_h \cdot X_{ai} \cdot \exp(-b_h \cdot t) \text{ or}$$

$$\log O_c = \log (f_{cv} \cdot (1 - f) \cdot b_h \cdot X_{ai}) - 2.3 \cdot b_h \cdot t \quad (8.12)$$

Equation 8.12 can be used to determine the constant  $b_h$  in the following way:

- (1) Determine the  $O_c$  values as a function of time in a digesting active sludge batch;
- (2) Plot the experimental data on semi log paper;
- (3) The best-fit straight line through the experimental points has a gradient of  $-2.3 \cdot b_h$ .

In Eq. (8.12) it is not taken into consideration that nitrification of mineralised ammonia may occur, which increases the observed oxygen uptake rate. The oxygen requirement for nitrification can be included by observing that the production of ammonium will be a fraction  $f_n$  of the digested sludge concentration. Hence, per unit mass of decayed active sludge, there is a production of  $f_n \cdot (1-f)$  g N, with an oxygen demand of  $4.57 \text{ mg O}_2 \cdot \text{mg}^{-1} \text{ N}$ . The OUR for nitrification can be expressed as:

$$\begin{aligned} O_n &= 4.57 \cdot f_n \cdot (1-f) \cdot r_d \\ &= 4.57 \cdot f_n \cdot (1-f) \cdot b_h \cdot X_{ai} \cdot \exp(-b_h \cdot t) \end{aligned}$$

Now the total OUR is given by:

$$\begin{aligned} O_t &= O_c + O_n \\ &= (f_{cv} + 4.57 \cdot f_n) \cdot (1-f) \cdot b_h \cdot X_{ai} \cdot \exp(-b_h \cdot t) \end{aligned} \quad (8.13)$$

Equation (8.13) shows that in the case of nitrification, the relationship between the logarithm of  $O_t$  and digestion time is also linear and has the same gradient as without nitrification. Therefore, the experimental procedure above for the determination of  $b_h$  is also valid when nitrification occurs. Alternatively, nitrification can also be inhibited through the addition of allylthiourea (ATU).

### 8.2.1.3 Variation of the nitrate concentration

When nitrification occurs in the sludge batch, there is a production of  $f_n \text{ NO}_3\text{-N}$  per unit mass of oxidised volatile sludge so that:

$$N_n - N_{ni} = f_n \cdot (X_{vi} - X_v) \quad (8.14)$$

$$N_{n\infty} - N_n = f_n \cdot (X_{vi} - X_{v,\infty}) \quad (8.14)$$

With  $N_{ni}$  = initial nitrate concentration

Combining Eqs. (8.10 and 8.14) one has:

$$N_{n\infty} - N_n = f_n \cdot (X_v - X_{v,\infty}) = f_n \cdot (1-f) \cdot X_{ai} \cdot \exp(-b_h \cdot t) \text{ or}$$

$$\log(N_{n\infty} - N_n) = \log(f_n \cdot (1-f) \cdot X_{ai}) - 2.3 \cdot b_h \cdot t \quad (8.15)$$

Equation (8.15) can be used to determine the value of  $b_h$ , using the procedure described in the sections above.

### 8.2.1.4 Variation of the alkalinity

The alkalinity of a sludge batch is influenced by ammonification and nitrification. In Section 4.1.2.2 it was established that the alkalinity production in the ammonification process is  $3.57 \text{ mg CaCO}_3 \cdot \text{mg}^{-1} \text{ N}$ , whereas there is a consumption of  $7.14 \text{ mg CaCO}_3 \cdot \text{mg}^{-1} \text{ N}$  during nitrification.

Hence, there is a net alkalinity reduction of  $3.57 \text{ mg CaCO}_3 \cdot \text{mg}^{-1} \text{ N}$  when nitrification of the mineralised ammonia occurs, so that:

$$\text{Alk}_i - \text{Alk} = 3.57 \cdot (\text{N}_n - \text{N}_{ni}) \quad (8.16)$$

And using Eq. (8.15):

$$\begin{aligned} \text{Alk} - \text{Alk}_\infty &= 3.57 \cdot f_n \cdot (1 - f) \cdot X_{ai} \cdot \exp(-b_h \cdot t) \text{ or} \\ \log(\text{Alk} - \text{Alk}_\infty) &= \log[3.57 \cdot f_n \cdot (1 - f) \cdot X_{ai}] - 2.3 \cdot b_h \cdot t \end{aligned} \quad (8.17)$$

Where  $\text{Alk}_\infty$  = alkalinity when digestion of the sludge batch is complete

Equation (8.17) is the fourth option that can be used to determine the decay constant  $b_h$ . The change in nitrate concentration and alkalinity can only be used when nitrification of the mineralised ammonia occurs. In practice this will usually be the case in an aerobic digester. If nitrification does not occur, both the ammonium concentration and the alkalinity will increase and this will lead to an increase of the pH. Under those circumstances part of the ammonium may be stripped (as gaseous  $\text{NH}_3$ ) by the aeration process, which reduces the alkalinity. Hence if nitrification does not occur, neither the concentrations of ammonium or nitrate nor the alkalinity can be used to determine the decay constant  $b_h$ .

Van Haandel et al (1985) applied the batch technique exemplified above to determine the decay constant in the range  $20\text{-}32^\circ\text{C}$ , observing the behaviour of the four parameters and obtained the following average result:

$$b_{hT} = 0.24 \cdot 1.04^{(T-20)} \text{ d}^{-1} \quad (20^\circ\text{C} < T < 32^\circ\text{C}) \quad (8.18a)$$

This result is very similar to the value obtained by Marais and Ekama (1976) for the range of temperatures from  $12$  to  $20^\circ$ :

$$b_{hT} = 0.24 \cdot 1.029^{(T-20)} \text{ d}^{-1} \quad (12^\circ\text{C} < T < 20^\circ\text{C}) \quad (8.18b)$$

It must be emphasised that in both cases the value of the constant was independent of the initial active sludge fraction, i.e. it does not depend on the sludge age of the process from which the excess sludge is taken.