6.2.3 Determination of the minimum concentration X_m

The minimum concentration X_m is determined by the condition that for this concentration the flux is equal to the limiting flux F_l (see Fig. 6.5c). Hence:

$$F_{X=Xm} = (F_v + F_u)_{X=Xm} = F_l \qquad (6.24)$$

After substituting Eqs. (6.13 and 6.18) in Eq.(6.24) and rearranging one has:

$$X_m \cdot \exp(-k \cdot X_m) = (X_r - X_m) \cdot (k \cdot X_l - 1) \cdot \exp(-k \cdot X_l) \qquad (6.25)$$

Equation (6.25) does not have an analytical solution, but the X values can be calculated as a function of X_r for any k-value using numerical methods. To represent the X_l and X_m graphically, it is convenient to construct an adimensional diagram, using $k \cdot X_r$ at the horizontal axis and $k \cdot X_l$ or $k \cdot X_m$ at the vertical axis. Figure 6.7 shows the $k \cdot X_l$ and $k \cdot X_m$ values as a function of $k \cdot X_r$, calculated with the aid of Eqs.(6.17 and 6.25) respectively. The value $k \cdot X_c = 4$ is also indicated on the horizontal axis. For $k \cdot X_c = 4$ one has $k \cdot X_l = k \cdot X_m = 2$, i.e. one has the critical sludge concentration displayed in Fig. 6.6.

Figure 6.7 has considerable practical utility: for any “pair” of inlet- and outlet concentrations X_t and X_r of a settler, it can immediately be determined which of the two functions of the settler is limiting: clarification or thickening. For $k \cdot X_r > k \cdot X_c = 4$ and $k \cdot X_m < k \cdot X_t < k \cdot X_l$, the limiting process is thickening. For all other cases the limiting process will be clarification. Later (Section 6.3) it will prove to be convenient to relate the inlet and outlet sludge concentrations X_t and X_r. This relationship, expressed in Eq. (6.10), is also indicated in Fig. 6.7.