

### 6.2.2 Determination of the critical concentration $X_c$

The tangential straight line passing through the point  $X_c$  at the horizontal axis is also described by (6.16), but the gradient of the line is now maximum as in Fig. 6.6. Hence:

$$(dm/dX) = (d^2F_v/dX^2)_{X=X_i} = 0 \text{ or } X_i = 2/k \text{ and } F_i = 2 \cdot v_0/(k \cdot e^2) \tag{6.20}$$

Where:

- $X_i$  = sludge concentration at the inflection point of curve  $F_v$
- $F_i$  = batch settling flux at the inflection point of  $F_v$
- $e$  = natural logarithm basis  $\approx 2.71$

The gradient of the straight line through  $(X_i, F_i)$  and  $(X_c, 0)$  is equal to the derivative of  $F_v$  at  $(X_i, F_i)$ :

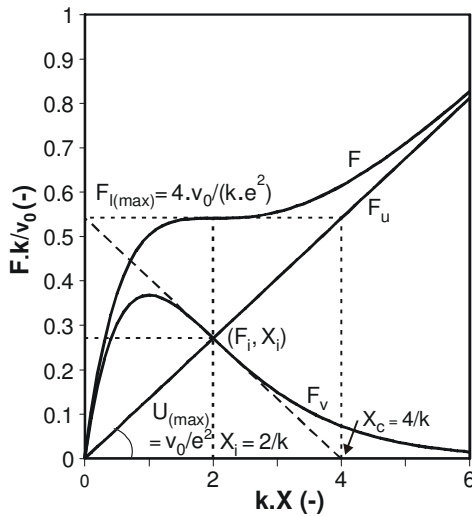
$$m = (dF_v/dX)_{X=2/k} = -v_0/e^2 \tag{6.21}$$

Hence, the straight line is given by:

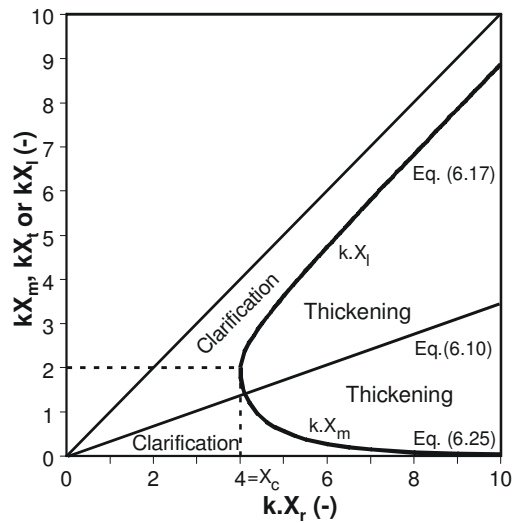
$$F - 2 \cdot v_0/(k \cdot e^2) = -v_0/e^2 \cdot (X - 2/k) \text{ or } F = -(v_0/e^2) \cdot (X - 4/k) \tag{6.22}$$

Now, the critical concentration can be determined, knowing that  $F = 0$  when  $X = X_c$ :

$$X_c = 4/k \tag{6.23}$$



**Figure 6.6**  
 $F_v$  and  $F_u$  curves as functions of the sludge concentration for  $X_r = X_c = 4/k$  (the coordinates are adimensional)



**Figure 6.7**  
 $k \cdot X_i$  and  $k \cdot X_m$  values as functions of  $k \cdot X_r$ , showing whether clarification or thickening is the limiting process