### 3.2.2 Model summary

Equation (3.43) completes the construction of the simplified model in the sense that now expressions have been derived for the division of the influent COD into fractions in the effluent ($mS_{te}$), in the excess sludge ($mS_{xv}$) and oxidised in the process ($mS_o$). For convenience the expressions are repeated below:

(1) Fraction of influent COD remaining in the liquid phase:

$$mS_{te} = f_{ns} \tag{3.18}$$

(2) Fraction of influent COD discharged with the excess sludge:

$$mS_{xv} = f_{cv} \cdot (1 - f_{ns} - f_{np}) \cdot (1 + f \cdot b_h \cdot R_s) \cdot C_r / R_s + f_{np} \tag{3.38}$$

(3) Fraction of influent COD oxidised in the process:

$$mS_o = (1 - f_{ns} - f_{np}) \cdot [(1 - f_{cv} \cdot Y) + f_{cv} \cdot (1 - f) \cdot b_h \cdot C_r] \tag{3.43}$$

It is interesting to note that the sum of the three fractions is identical to unity:

$$mS_{te} + mS_{xv} + mS_o \equiv 1 \tag{3.44}$$

The basic equations forming the simplified model are brought together in Table 3.4. It can be seen that in all equations the hydraulic retention time is present, which may give the (erroneous) impression that this parameter has a fundamental importance.

To demonstrate that the liquid retention time is of no importance for modelling of basic activated sludge process behaviour, the equations can be rewritten in the form of mass equations, in which the hydraulic retention time does not play a role. In Table 3.5 the mass equations are presented. In these equations, rather than the concentrations, the masses are considered as variables. Thus one has for example:

$$\begin{aligned} mX_v &= MX_v/MS_{ti} = V_r \cdot X_v/(Q_i \cdot S_{ti}) = R_h \cdot X_v/S_{ti} \\ &= (1 - f_{ns} - f_{np}) \cdot (1 + f \cdot b_h \cdot R_s) \cdot C_r + f_{np} \cdot R_s / f_{cv} \end{aligned} \tag{3.50}$$

$mX_v$ = mass of organic sludge present in the system per unit mass daily applied COD
$MX_v$ = volatile sludge mass present in the system (kg VSS)
$MS_{ti}$ = daily applied COD mass (kg $COD.d^{-1}$)

**Table 3.4 Basic equations of the activated sludge system**

| Par. | Equation | No. |
|---|---|---|
| $C_r$ | $= Y \cdot R_s/(1+b_h \cdot R_s)$ | (3.30) |
| $S_{bi}$ | $= (1 - f_{ns} - f_{np}) \cdot S_{ti}$ | (3.3) |
| $S_{te}$ | $= f_{ns} \cdot S_{ti}$ | (3.1a) |
| $X_i$ | $= (f_{np} \cdot R_s/f_{cv}) \cdot S_{ti}/R_h$ | (3.21) |
| $X_a$ | $= C_r \cdot S_{bi}/R_h$<br>$= [(1 - f_{ns} - f_{np}) \cdot Y \cdot R_s/(1 + b_h \cdot R_s)] \cdot S_{ti}/R_h$ | (3.28) |
| $X_e$ | $= f \cdot b_h \cdot R_s \cdot C_r \cdot S_{bi}/R_h$<br>$= f \cdot b_h \cdot (1 - f_{ns} - f_{np}) \cdot Y \cdot R_s/(1 + b_h \cdot R_s)] \cdot S_{ti}/R_h$ | (3.34) |
| $X_v$ | $= [(1 - f_{ns} - f_{np}) \cdot (1 + f \cdot b_h \cdot R_s) \cdot C_r + f_{np} \cdot R_s/f_{cv}] \cdot S_{ti}/R_h$<br>$= [(1 - f_{ns} - f_{np}) \cdot (1 + f \cdot b_h \cdot R_s) \cdot Y \cdot R_s/(1 + b_h \cdot R_s) + f_{np} \cdot R_s/f_{cv}] \cdot S_{ti}/R_h$ | (3.35) |
| $O_c$ | $= (1 - f_{ns} - f_{np}) \cdot (1 - f_{cv} \cdot Y + (1-f) \cdot f_{cv} \cdot b_h \cdot (Y \cdot R_s/(1 + b_h \cdot R_s)) \cdot S_{ti}/R_h$<br>$= (1 - f_{cv} \cdot Y + (1 - f) \cdot f_{cv} \cdot b_h \cdot C_r) \cdot S_{bi}/R_h$ | (3.42) |

**Table 3.5 Mass equations of the activated sludge system**

| Par. | Equations | | No. |
|---|---|---|---|
| $mS_{bi}$ | $= MS_{bi}/MS_{ti}$ | $= (Q_i \cdot S_{bi})/(Q_i \cdot S_{ti})$<br>$= (1 - f_{ns} - f_{np})$ | (3.45) |
| $mS_{te}$ | $= MS_{te}/MS_{ti}$ | $= (Q_i \cdot S_{nsi})/(Q_i \cdot S_{ti})$<br>$= f_{ns}$ | (3.46) |
| $mX_i$ | $= MX_i/MS_{ti}$ | $= (V_r \cdot X_i)/(Q_i \cdot S_{ti})$<br>$= f_{np} \cdot R_s/f_{cv}$ | (3.47) |
| $mX_a$ | $= MX_a/MS_{ti}$ | $= (V_r \cdot X_a)/(Q_i \cdot S_{ti})$<br>$= (1 - f_{ns} - f_{np}) \cdot C_r$ | (3.48) |
| $mX_e$ | $= MX_e/MS_{ti}$ | $= (V_r \cdot X_e)/(Q_i \cdot S_{ti})$<br>$= (1 - f_{ns} - f_{np}) \cdot C_r \cdot f \cdot b_h \cdot R_s$ | (3.49) |
| $mX_v$ | $= MX_v/MS_{ti}$<br>$= mX_i+mX_a+mX_e$ | $= V_r \cdot (X_e+X_a+X_i)/(Q_i \cdot S_{ti})$<br>$= (1 - f_{ns} - f_{np}) \cdot (1 + f \cdot b_h \cdot R_s) \cdot C_r + f_{np} \cdot R_s/f_{cv}$ | (3.50) |
| $mX_t$ | $= MX_t/MS_{ti}$ | $= (V_r \cdot X_t)/(Q_i \cdot S_{ti})$<br>$= mX_v/f_v$ | (3.51) |
| $mS_o$ | $= MO_c/MS_{ti}$ | $= V_r \cdot O_c/(Q_i \cdot S_{ti})$<br>$= (1 - f_{ns} - f_{np}) \cdot (1 - f_{cv} \cdot Y + f_{cv} \cdot (1-f) \cdot b_h \cdot C_r)$ | (3.52) |
| $mS_{xv}$ | $= (f_{cv} \cdot MX_v/R_s)/MS_{ti}$ | $= (f_{cv} \cdot V_r \cdot X_v/R_s)/(Q_i \cdot S_{ti})$<br>$= f_{cv} \cdot (1 - f_{ns} - f_{np}) \cdot (1 + f \cdot b_h \cdot R_s) \cdot C_r/R_s + f_{np}$ | (3.38) |
| $mS_{ti}$ | $= mS_{te} + mS_o + mS_{xv}$ | $= 1$ | (3.44) |