3.2.1 Model development

The first important step towards modelling the activated sludge system is to simplify the system as much as possible. First an ideal activated sludge system will be considered with one completely mixed reactor, operating under constant flow and load conditions.

The term ideal indicates that (l) all the biodegradable organic material is effectively metabolised in the process and (2) the settler is a perfect liquid-solid separator in the sense that there are no suspended solids in the effluent and that the sludge hold-up in the settler is negligible in relation to the sludge mass in the biological reactor.

The term constant flow and load implies that the excess sludge and the influent both have a constant flow and composition. A fixed rate of sludge discharge is necessary to establish a constant sludge mass in the process, characterised by the fact that the sludge growth rate is equal to the withdrawal rate due to excess sludge wastage. It is also assumed that the sludge is discharged directly from the reactor and that the composition of the excess sludge is equal to that of the mixed liquor in the reactor.

Later in this chapter a general model will be discussed that can also be applied when the above restrictions do not apply, resulting in a much more complex process description. In Fig. 3.3 the processes that form the basis of the simplified model for the activated sludge system are represented.

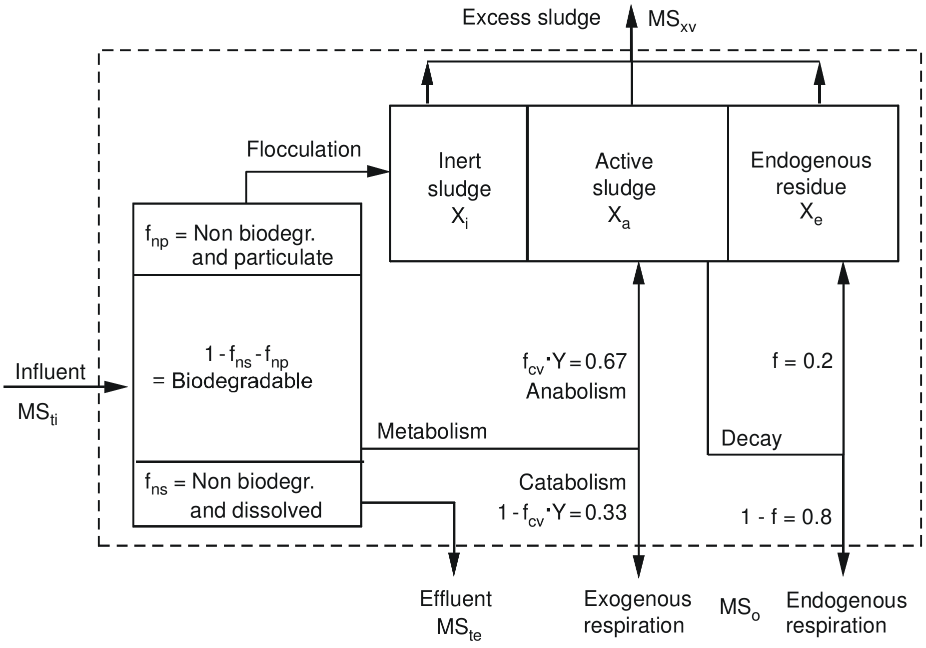

Figure 3.3 Overview of the processes that develop in an ideal activated sludge system

When a waste water containing organic material is placed in contact with an activated sludge mass under aerated conditions, the following processes will occur:

(a) Metabolism

The biodegradable organic material in the influent is removed from the liquid phase and metabolised by the sludge. It was observed in Chapter 2 that this process leads to both sludge growth (anabolism) and oxygen consumption (catabolism)

(b) Decay

It is postulated that sludge decay is independent of metabolic processes and that part of the decayed active sludge is oxidised to inorganic compounds, whereas the remainder accumulates in the reactor as endogenous residue until it is discharged with the excess sludge. The oxygen consumption due to oxidation of active sludge is called endogenous respiration, to distinguish it from the oxidation of influent organic material, which is called exogenous respiration. The independence of endogenous and exogenous respiration will be demonstrated in Chapter 8.

(c) Bioflocculation

The particulate non-biodegradable organic material in the influent is not affected by the metabolic activity of the sludge, but is removed physically from the liquid phase by flocculation. The flocculated material constitutes the inert sludge fraction. In the model of Fig. 3.3 the biodegradable fractions and the particulate non-biodegradable fractions are removed from the liquid phase, but the fourth fraction, dissolved non-biodegradable organic material is not affected in any way by the activated sludge system and is discharged without modifications into the effluent.

3.2.1.1 Definition of sludge age

Having defined the conditions to formulate the simplified model, the most important operational variable will now be defined: the sludge age R_s. This parameter indicates the average retention time of the sludge in the system and is defined as the ratio between the sludge mass present in the system and the daily sludge mass discharged from it. Using the model of Fig. 3.3 and assuming hydraulic sludge wasting (i.e. excess sludge discharge from the aeration tank) one has:

$$\begin{aligned} R_s &= MX_t/ME_t \\ &= V_r \cdot X_t/(q \cdot X_t) = V_r/q \end{aligned} \tag{3.15}$$

Where:

R_s = sludge age (d)
MX_t = sludge mass in the system (kg TSS)
ME_t = daily discharge of excess sludge (kg $TSS.d^{-1}$).

Equation (3.15) can also be written in another way:

$$q = V_r/R_s \tag{3.16}$$

Equation (3.16) expresses that the flow of excess sludge, when discharged from the biological reactor, is a fraction $1/R_s$ of the reactor volume, i.e. over a period of R_s days the volume of wasted sludge is equal to the reactor volume. The sludge age is independent of the liquid retention time R_h. This parameter is defined as the ratio between the reactor volume and the influent flow:

$$R_h = V_r/Q_i \tag{3.17}$$

Using the definitions for the sludge age and the liquid retention time, it is now possible to derive expressions to predict the values of the COD fractions mS_{te}, mS_{xv} and mS_o, which is the objective of the simplified model for the activated sludge system.

3.2.1.2 COD fraction discharged with the effluent

In the ideal activated sludge system, the effluent COD and the COD of the liquid phase of the mixed liquor are both equal to the non-biodegradable dissolved organic material in the influent, S_{nsi}. From the definition of S_{nsi} in Eq. (3.2), one has:

$$mS_{te} = S_{te}/S_{ti} = S_{nsi}/S_{ti} = f_{ns} \qquad (3.18)$$

Hence the simplified model predicts a constant effluent COD, independent of the sludge age or the liquid retention time and equal to the non biodegradable and dissolved COD fraction in the influent.

3.2.1.3 COD fraction in the excess sludge

The determination of this fraction is more complicated and requires derivation of expressions for the three fractions that compose the organic sludge: inert sludge, active sludge and endogenous residue.

(a) The inert sludge X_i

The inert sludge concentration can be calculated easily from a simple mass balance using Fig. 3.2. The inert sludge is generated by flocculation of the particulate and non-biodegradable material in the influent and is discharged in the excess sludge. Since the inert material is not affected by biochemical processes, the influent mass flow must be equal to the mass flow in the excess sludge, so that:

$$Q_i \cdot X_{ii} = q \cdot X_i \qquad (3.19)$$

Where X_{ii} = concentration of non-biodegradable suspended solids in the influent
(mg VSS.1^{-1}).

The concentration X_{ii} can be correlated to the particulate and non-biodegradable COD fraction in the influent, by recognising the proportionality between COD and volatile suspended solids (f_{cv} = 1.5 mg COD.mg^{-1} VSS):

$$X_{ii} = S_{npi}/f_{cv} = (f_{np}/f_{cv}) \cdot S_{ti} \qquad (3.20)$$

Now, using Eq. (3.20) in Eq. (3.19) leads to:

$$\begin{aligned} X_i &= (f_{np}/f_{cv}) \cdot (Q_i/q) \cdot S_{ti} \\ &= (f_{np}/f_{cv}) \cdot (R_s/R_h) \cdot S_{ti} \end{aligned} \qquad (3.21)$$

(b) The active sludge X_a

As can be noted in Fig. 3.3, the active sludge concentration is affected by three factors: (l) sludge growth due to synthesis, (2) decay and (3) sludge wastage. The variation of the active sludge concentration can be expressed as the sum of these three processes:

$dX_a/dt = (dX_a/dt)_g + (dX_a/dt)_d + (dX_a/dt)_e$ (3.22)

Where:

X_a = active sludge concentration (mg VSS.1^{-1})
(dX_a/dt) = rate of change of the active sludge concentration (mg VSS.l^{-1}.d^{-1})
$(dX_a/dt)_g$ = growth rate due to synthesis (mg VSS.l^{-1}.d^{-1})
$(dX_a/dt)_d$ = decay rate of active sludge (mg VSS.l^{-1}.d^{-1})
$(dX_a/dt)_e$ = wastage rate of active sludge in excess sludge (mg VSS.l^{-1}.d^{-1})

Under steady state conditions, the active sludge concentration does not change with time so that:

$dX_a/dt = 0 = (dX_a/dt)_g + (dX_a/dt)_d + (dX_a/dt)_e$ (3.23)

The active sludge growth rate is proportional to the utilisation rate of biodegradable material, with a yield of Y kg active sludge synthesised per unit mass of utilised COD. In the ideal activated sludge system the utilisation rate of biodegradable material will be equal to the feed rate and can be calculated as:

$V_r \cdot (dS_{bi}/dt)_u = V_r \cdot r_u = Q_i \cdot S_{bi}$ or $r_u = S_{bi} \cdot Q_i/V_r = S_{bi}/R_h$ (3.24)

Where $r_u = (dS_{bi}/dt)_u$ = utilisation rate of biodegradable material (mg COD.l^{-1}.d^{-1})
Now the growth rate of active sludge can be calculated as:

$r_c = (dX_a/dt)_c = Y \cdot r_u = Y \cdot S_{bi}/R_h$ (3.25)

Where Y = yield coefficient for active sludge (mg VSS.mg^{-1} COD)

In Chapter 8 it will be shown that the decay rate of active sludge can be expressed as a first order process with respect to the active sludge concentration:

$r_d = (dX_a/dt)_d = -b_h \cdot X_a$ (3.26)

Where b_h = decay constant for active sludge (d^{-1})

The rate at which the active sludge concentration decreases due to sludge wastage can by definition be expressed as:

R_s = (active sludge mass)/(wastage rate of active sludge)
$= V_r \cdot X_a/(V_r \cdot (-dX_a/dt)_e)$

Hence:

$r_e = (dX_a/dt)_e = -X_a/R_s$ (3.27)

Substituting Eqs. (3.25 to 3.27) in Eq. (3.23), the following expression is obtained for the active sludge concentration:

$Y \cdot S_{bi}/R_h - b_h \cdot X_a - X_a/R_s = 0$ or

$X_a = [Y \cdot R_s/(1 + b_h \cdot R_s)] \cdot S_{bi}/R_h$ (3.28)

Now by using Eq. (3.3) to substitute for S_{bi} one has:

$$X_a = [(1 - f_{ns} - f_{np})\cdot Y\cdot R_s/(1 + b_h\cdot R_s)] \cdot S_{ti}/R_h$$
$$= (1 - f_{ns} - f_{np})\cdot C_r\cdot S_{ti}/R_h \quad (3.29)$$

Where

$$C_r = Y\cdot R_s/(1 + b_h\cdot R_s) \quad (3.30)$$

C_r represents the active sludge mass present in the system per unit mass daily applied biodegradable organic material. The inverse of C_r is the COD utilisation rate per unit mass active sludge or the specific utilisation rate of the organic material, discussed in Section 3.2.3.5.

(c) The endogenous residue X_e

Once again, under steady state conditions the concentration of the endogenous residue does not change with time. Thus the concentration can be calculated from the fact that the production rate is equal to the withdrawal rate:

$$(dX_e/dt) = 0 = (dX_e/dt)_d + (dX_e/dt)_e \quad (3.31)$$

Where (dX_e/dt) = rate of change of endogenous residue concentration. Indices “d” and “e” refer to active sludge decay and excess sludge wastage respectively.

Upon decay of active sludge, a constant fraction is transformed into endogenous residue, whereas the remainder is oxidised. Hence, the production rate of endogenous residue is proportional to the active sludge decay rate and the proportionality constant is equal to the fraction of decayed active sludge remaining as endogenous residue. Hence:

$$(dX_e/dt)_d = -f\cdot(dX_a/dt)_d = f\cdot b_h\cdot X_a \quad (3.32)$$

Where f = fraction of decayed active sludge transformed into endogenous residue.

The rate of decrease of endogenous residue concentration due to sludge wastage is calculated by using Eq.(3.27):

$$(dX_e/dt)_e = -X_e/R_s \quad (3.33)$$

Substituting Eqs. (3.32 and 3.33) in Eq. (3.31) one has:

$f\cdot b_h\cdot X_a - X_e/R_s = 0$ or

$$X_e = f\cdot b_h\cdot R_s\cdot X_a \quad (3.34)$$

(d) The organic sludge

The organic or volatile sludge concentration is equal to the sum of the three fractions: inert, active and endogenous residue. Hence, from Eqs. (3.21, 3.29 and 3.34) one has:

$$X_v = X_a + X_e + X_i = [(1 - f_{ns} - f_{np})\cdot C_r\cdot(1 + f\cdot b_h\cdot R_s) + f_{np}\cdot R_s/f_{cv}]\cdot S_{ti}/R_h \quad (3.35)$$

The expression for the organic sludge concentration is particularly important because this parameter can be determined experimentally, allowing the possibility to verify if the calculated theoretical concentration is equal to the actual value. After having derived an expression for the organic sludge concentration, it becomes a simple matter to calculate the sludge mass in the reactor and the sludge production. The product of the volatile sludge concentration and the reactor volume V_r gives the sludge mass MX_v. For a particular sludge age R_s, the sludge production rate will be a fraction $1/R_s$ of the existing sludge mass.

$$MX_v = V_r \cdot X_v = [(1 - f_{ns} - f_{np}) \cdot (1 + f \cdot b_h \cdot R_s) \cdot C_r + f_{np} \cdot R_s / f_{cv}] \cdot Q_i \cdot S_{ti} \text{ and} \tag{3.36}$$

$$ME_v = V_r \cdot X_v / R_s = [(1 - f_{ns} - f_{np}) \cdot (1 + f \cdot b_h \cdot R_s) \cdot C_r / R_s + f_{np} / f_{cv}] \cdot MS_{ti} \tag{3.37}$$

Where:

MX_v = organic sludge mass in the system (kg VSS)
ME_v = daily organic sludge production (kg $VSS \cdot d^{-1}$)

Having established an expression for the sludge production rate and knowing that there is a proportionality between the organic sludge mass and its COD, it is now possible to calculate the fraction of the influent COD that is wasted as excess sludge:

$$\begin{aligned} mS_{xv} &= ME_v / MS_{ti} = f_{cv} \cdot (V_r \cdot X_v / R_s) / (Q_i \cdot S_{ti}) \\ &= f_{cv} \cdot (1 - f_{ns} - f_{np}) \cdot (1 + f \cdot b_h \cdot R_s) \cdot C_r / R_s + f_{np} \end{aligned} \tag{3.38}$$

3.2.1.4 COD fraction oxidised for respiration

Oxygen is consumed for both exogenous and endogenous respiration. The oxygen uptake rate (OUR) due to exogenous respiration O_{ex} is determined from Fig. 3.3, where it is shown that upon metabolism of 1 gram of COD, there will be a production of active sludge of Y gram of VSS with a COD value of $f_{cv} \cdot Y$ gram COD. Hence the remaining fraction of $(1-f_{cv} \cdot Y)$ gram COD will be oxidised and for that oxidation, by definition, an oxygen mass of $(1-f_{cv} \cdot Y)$ gram O_2 is required. Hence the exogenous consumption rate can be expressed as:

$$\begin{aligned} O_{ex} &= (1 - f_{cv} \cdot Y) \cdot r_u \\ &= (1 - f_{cv} \cdot Y) \cdot S_{bi} / R_h \end{aligned} \tag{3.39}$$

The OUR for endogenous respiration O_{en} is calculated from the oxidation rate of decayed activated sludge, which is the difference between the decay rate and the production rate of endogenous residue:

$$\begin{aligned} r_o &= (dX_a/dt)_d - (dX_e/dt)_d \\ &= b_h \cdot X_a - f \cdot b_h \cdot X_a = (1 - f) \cdot b_h \cdot X_a \end{aligned} \tag{3.40}$$

Where r_o = oxidation rate of decayed active sludge

Again using the proportionality constant f_{cv} the endogenous respiration rate can be calculated:

$$\begin{aligned} O_{en} &= f_{cv} \cdot r_o \\ &= f_{cv} \cdot (1 - f) \cdot b_h \cdot X_a \end{aligned} \tag{3.41}$$

The total OUR for the oxidation of organic material is equal to the sum of the values for exogenous and for endogenous respiration:

$O_c \quad = O_{ex} + O_{en}$

Using Eqs. (3.3 and 3.29) to substitute for S_{bi} and X_a leads to:

$$O_c = (1 - f_{ns} - f_{np})\cdot(1 - f_{cv}\cdot Y + f_{cv}\cdot(1 - f)\cdot b_h\cdot C_r)\cdot S_{ti}/R_h \quad (3.42)$$

The influent COD fraction that is oxidised in the activated sludge system is now expressed as:

$$\begin{aligned} mS_o &= MO_c/MS_{ti} = (V_r\cdot O_c)/(Q_i\cdot S_{ti}) \\ &= (1 - f_{ns} - f_{np})\cdot[(1 - f_{cv}\cdot Y) + f_{cv}\cdot(1 - f)\cdot b_h\cdot C_r] \quad (3.43) \end{aligned}$$